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Heavy quarkonia from classical SU(3) Yang-Mills configurations

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Abstract. A generalized Cho-Faddeev-Niemi ansatz for $SU(3)$ Yang-Mills is investigated. The corresponding classical field equations are solved for its simplest parametrization. From these solutions it is possible to define a confining non-relativistic central potential used to study heavy quarkonia. The associated spectra reproduces the experimental spectra with an error of less than 3% for charmonium and 1% for bottomonium. Moreover, the recently discovered new charmonium states can be accomodate in the spectra, keeping the same level of precision. The leptonic widths show good agreement with the recent measurements. The charmonium and bottomonium $E1$ electromagnetic transitions widths are computed and compared with the experimental values.

PACS. 12.39.Pn Potential models – 12.38.Lg Other nonperturbative calculations – 12.38.-t Quantum chromodynamics

1 Introduction

Charmonium and bottomonium are essentially non relativistic systems. Their dynamics can be understood in terms of a potential. Ideally, we would be able to derive such a potential directly from QCD. However, the confining potentials currently used to describe such systems, although related to the fundamental theory, are not directly derived from QCD.

In this paper, we report on a class of classical solutions of the Yang-Mills theory, computed after the introduction of a generalized Cho-Faddeev-Niemi [1,2] ansatz for the gluon field. The solutions allow a definition of a confining potential which is then used to study charmonium and bottomonium spectra, their leptonic decays and charmonium electromagnetic transitions. For interquark distance between 0.2 fm to 1 fm, the new potential is essentially the lattice singlet potential. For larger distances it grows exponentially with the interquark distance and for smaller distances it is Coulomb like. Concerning the spectra, the new potential is able to reproduce charmonium and bottomonium spectra with an error of less than 3% and 1%, respectively, including the recently discovered new charmonium states $X(3872)$, $X(3940)$, $Y(3940)$, $Z(3940)$ and $Y(4260)$. If the spectra is reasonably well described, the hyperfine splittings ${}^3S_1$-${}^1S_0$ turn out to be too small. This can be due to a missing scalar confining potential or the need for including contributions from other configurations, namely those related with the short-distance behaviour of the theory. As for the leptonic decays and the charmonium electromagnetic transitions, the new potential describes well the experimental data. Part of this work is described in [3].

2 Classical configuration and a heavy quark potential

Following a procedure suggested in [4], we introduce a real field n^a and we require the field n^a to be a covariant constant

$$D_\mu n^a = \partial_\mu n^a + g f_{abc} n^b A^c_\mu = 0\,. \qquad (1)$$

Multiplying this equation by n^a, we realize that n^a can be unitary. Defining the color projected field C_μ such that $A^a_\mu = n^a C_\mu + X^a_\mu$, with $n^a X^a_\mu = 0$ and replacing this definition in (1), X^a_μ can be related to n^a:

$$A^a_\mu = n^a\, C_\mu + \frac{3}{2g} f_{abc} n^b \partial_\mu n^c + Y^a_\mu\,, \quad n^a Y^a_\mu = 0\,. \qquad (2)$$

For the simplest parametrization of $n^a = \delta^{a1}\sin\theta + \delta^{a2}\cos\theta$ and for the simplest gluon configuration (with $Y_\mu = C_\mu = 0$), the classical field equations in Landau gauge become

$$\partial^\mu \partial_\mu \theta = 0\,, \qquad (3)$$

with $gA^a_\mu = -\delta^{a3}\partial_\mu\theta$. Note that there are no boundary conditions for θ. Equation (3) can be solved by the usual

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method of separation of variables. The solution considered here is

$$A_0^3 = \Lambda\left(e^{\Lambda t} - b_T e^{-\Lambda t}\right) V_0(r), \tag{4}$$

$$\boldsymbol{A}^3 = -\left(e^{\Lambda t} + b_T e^{-\Lambda t}\right) \nabla V_0(r), \tag{5}$$

$$V_0(r) = A\frac{\sinh(\Lambda r)}{r} + B\frac{e^{-\Lambda R}}{r}. \tag{6}$$

Assuming that the spatial part of A_0^3 can be identified with a non relativistic potential for heavy quarkonia (see [3] for details), the squared difference between V_0 and the lattice singlet potential integrated between 0.2–1 fm was minimized to define the various parameters; for r in MeV^{-1}, $A = 11.254$, $B = -0.701$, $\Lambda = 228.026$ MeV. The difference between V_0 and the singlet potential is less than 50 MeV for $r \in [0.2\text{–}1]$ fm. From now on, we assume that V_0 describes the interaction between the heavy quarks and we include perturbatively a spin-dependent contribution supposing that the potential is of pure vectorial type.

2.1 Charmonium and bottomonium spectra

The charm quark mass, $m_c = 1870$ MeV, was adjusted to reproduce the experimental value of the $\chi_{c2}(1P) - J/\psi(1S)$ mass difference, whereas the bottom quark mass, $m_b = 4185.25$ MeV, was defined to reproduce the experimental value for the $\Upsilon(2S) - \Upsilon(1S)$ mass difference.

In table 1 we report the low-lying charmonium states. The masses were shifted ($\Delta_{cc} = -3223$ MeV) to reproduce the $J/\psi(1S)$ experimental value ($M = E_{nr} + 2m_Q + \Delta_{QQ}$). In what concerns the spectrum, there is good agreement between the theoretical and the measured values. The only particles which do not fit well in the spectrum are $\psi(4040)$ and $\psi(4415)$. About these two states, the experimental information is scarse and the particle data book comments that the "interpretation of these states as a single resonance is unclear because of the expectation of substantial threshold effects in this energy region". Curiously, both particle masses are essentially the sum of J/ψ with $J^{PC} = 0^{++}$ known mesons.

As for the new charmonium states, the theoretical predictions which are close to the experimental values are: $Z(3930)$, mass of $3929 \pm 5 \pm 2$ MeV, $J = 2$, is a 1^3F_2 state with 3932 MeV or a 2^3P_2 state with 4048 MeV; $X(3940)$, mass of $3942 \pm 11 \pm 13$ MeV, and $Y(3940)$, mass of $3943 \pm 11 \pm 13$ MeV, can be any of the following states 2^3P_0, 3831 MeV, 2^3P_1, 3938 MeV, or 2^1P_1, 3959 MeV; $Y(4260)$, mass of 4260 MeV, and with quantum numbers 1^{--} as established by the experience, can be identified as a 3^3S_1 state with 4164 MeV. For charmonium $X(3872)$, mass of 3871.2 ± 0.5 MeV, the experimental data favours the following quantum numbers $J^{PC} = 1^{++}, 2^{-+}$. For the potential considered here, the closest states with such quantum numbers are 2^3P_1, 3938 MeV, and 2^3P_0, 3832 MeV.

In table 2 we report the low-lying bottomonium states. The masses were shifted ($\Delta_{bb} = -1136$ MeV) to reproduce the $\Upsilon(1S)$ experimental value. The discrepancy between the predicted and experimental value for $\Upsilon(4S)$ is too large when compared with other results in the table. It is more natural to assign the $\Upsilon(4S)$ to a 2^3D_1 state, just taking into account the predicted masses.

In conclusion, the non-relativistic potential V_0 is able to explain the charmonium spectrum with an error of less than 3% (~ 100 MeV) and the bottomonium spectrum with an error of less than 1.5% (~ 100 MeV), *i.e.*, the level of accuracy achieved is within other potential model calculations (see, for example, [6] and references therein).

Table 1. Charmonium spectra. The right columns show the difference, in MeV, between the theoretical prediction and the experimental values taken from [5].

Theor. (MeV)	Expt. (MeV)		Theor. (MeV)	Expt. (MeV)	
$J^{PC} = 0^{-+}$			$[1P]$		
3075	$\eta_c(1S)$	95	3556	$\chi_{c2}(1P)$	0
3632	$\eta'_c(2S)$	−5	3462	$\chi_{c1}(1P)$	−49
4135	—	—	3372	$\chi_{c0}(1P)$	−43
			3478	$h_c(1P)$	−48
$J^{PC} = 1^{--}$ $[^3S_1]$			$J^{PC} = 1^{--}$ $[^3D_1]$		
3097	$J/\psi(1S)$	0			
3659	$\psi(2S)$	−27	3688	$\psi(3770)$	−83
4164	$\Upsilon(4260)$	−95	4155	—	—
$M[J/\psi(1S) - \eta_c(1S)] = 22\vert_{\text{Theo}} = 117\vert_{\text{Exp}}$ MeV					
$M[\psi(2S) - \eta'_c(2S)] = 26\vert_{\text{Theo}} = 48\vert_{\text{Exp}}$ MeV					

Table 2. Bottomonium spectra. The right columns report the difference, in MeV, between theory and experimental values.

Theor. (MeV)	Expt. (MeV)		Theor. (MeV)	Expt. (MeV)	
$J^{PC} = 0^{-+}$			$J^{PC} = 1^{--}$ $[^3S_1]$		
9457	$\eta_b(1S)$	157	9460	$\Upsilon(1S)$	0
10018	$\eta'_b(2S)$	—	10023	$\Upsilon(2S)$	0
10380	—	—	10385	$\Upsilon(3S)$	30
10721	—	—	10727	$\Upsilon(4S)$	148
11059	—	—	11065	$\Upsilon(11020)$	46
$[1P]$			$[2P]$		
9983	$\chi_{b2}(1P)$	71	10326	$\chi_{b2}(2P)$	57
9941	$\chi_{b1}(1P)$	48	10283	$\chi_{b1}(2P)$	28
9894	$\chi_{b0}(1P)$	35	10234	$\chi_{b0}(2P)$	2
9955	$h_b(1P)$	—	10296	$h_b(2P)$	—
$[^3D_1]$			$[^3D_2]$		
10159	—	—	10179	$\Upsilon(1D)$	18
10476	$\Upsilon(4S)$ (?)	103			
10796	$\Upsilon(10860)$	69			

Table 3. Charmonium and bottomonium leptonic widths in keV. The experimental figures are from [5]. The limit for $Y(4260)$ is from [7].

		Theor.	Expt.		Theor.	Expt.
2^3S_1	$\psi(2S)$	2.84	2.48(6)	$\Upsilon(2S)$	0.426	0.612(11)
3^3S_1	$Y(4260)$	2.16	< 0.40	$\Upsilon(3S)$	0.356	0.443(8)
4^3S_1	—	—	—	$\Upsilon(4S)$	0.335	0.272(29)
5^3S_1	—	—	—	$\Upsilon(5S)$	0.311	0.130(30)

2.2 Leptonic decays of charmonium and bottomonium

For the leptonic decays we follow the van Royen-Weisskopf [8] approach and assume that QCD corrections factorize in the calculation of the widths, *i.e.*

$$\Gamma_{e^+e^-}(^3S_1) = \frac{4\, e_q^2\, \alpha^2}{M^2}\, |R_{nS}(0)|^2 \,, \tag{7}$$

$$\Gamma_{e^+e^-}(^3D_1) = \frac{25\, e_q^2\, \alpha^2}{2\, m_q^4\, M^2}\, \left|R_{nD}^{(2)}(0)\right|^2 \,, \tag{8}$$

times QCD corrections; e_q is the quark electric charge, α the fine structure constant, m_q the quark mass, M the meson mass, $R_{nS}(0)$ the S-wave meson radial wave function at the origin and $R_{nD}^{(2)}(0)$ the D-wave meson second derivative of the radial wave function at the origin. In order to avoid estimating the QCD corrections, in table 3 we report the theoretical widths computed relatively to $J/\psi(1S)$ for charmonium and $\Upsilon(1S)$ for bottomonium.

For charmonium, $\Gamma_{e^+e^-}$ for $\psi(2S)$ is slightly larger than the experimental value. For bottomonium, the witdh for the $\Upsilon(2S)$ is below the experimental value although the witdhs for $\Upsilon(3S)$ and $\Upsilon(4S)$ agree with the experimental figure within two standard deviations. For $\Upsilon(5S)$, the theoretical prediction is a factor of 3 higher than the experimental figure. The theoretical prediction for $\Gamma_{e^+e^-}$ for $Y(4260)$ is much larger than the experimental limit. However, the 3^3S_1 and 2^3D_1 states are almost degenerate in mass and we expect a large mixing for the leptonic width to become compatible with the experimental value. A similar situation is seen in $\Psi(3770)$ and $\Psi(2S)$; see [3] for details.

2.3 E1 electromagnetic transitions

In relation to the computation of the $E1$ electromagnetic charmonium transitions we follow [9]. Table 4 reports the theoretical estimates of the decay widths known experimentaly. Overall, the agreement between theory and experiment is good. The exception being the transitions involving the scalar meson $\chi_{c0}(1P)$. Remember that for the meson spectra, the larger deviations from the experimental numbers occured for the scalar mesons.

Given the good agreement with the known experimental radiative transitions of the charmonium, we have computed the $E1$ electromagnetic widths for the theoretical states which can describe the new charmonium states. In table 5 we report those widths which are larger than $\sim 40\,\mathrm{keV}$. In principle, the electromagnetic transitions allows us to distinguish not only the various theoretical models but also the different meson states.

To conclude, in table 6 we report the bottomonium $E1$ electromagnetic transitions.

Table 4. $E1$ electromagnetic charmonium widths in keV. In the calculation for the meson mass it was used the experimental one. Experimental values are from [5].

		Theor.	Expt.
$\psi(2S) \longrightarrow$	$\chi_{c2}(1P)+\gamma$	30	27 ± 2
	$\chi_{c1}(1P)+\gamma$	43	29 ± 2
	$\chi_{c0}(1P)+\gamma$	51	31 ± 2
$\chi_{c2}(1P) \longrightarrow$	$J/\psi(1S)+\gamma$	414	416 ± 32
$\chi_{c1}(1P) \longrightarrow$	$J/\psi(1S)+\gamma$	308	317 ± 25
$\chi_{c0}(1P) \longrightarrow$	$J/\psi(1S)+\gamma$	146	135 ± 15
$\psi(3770)(1^3D_1) \longrightarrow$	$\chi_{c2}(1P)+\gamma$	4	< 21
	$\chi_{c1}(1P)+\gamma$	110	75 ± 18
	$\chi_{c0}(1P)+\gamma$	359	172 ± 30

Table 5. $E1$ electromagnetic charmonium widths, in keV, for the new charmonium states.

		E_γ	Γ
$X(3872)[2^3P_1] \longrightarrow$	$J/\psi(1S)+\gamma$	697	48
	$\psi(2S)+\gamma$	181	63
$X(3940)$, $Y(3940)$			
2^3P_1 or $2^3P_0 \longrightarrow$	$J/\psi(1S)+\gamma$	755	61
	$\psi(2S)+\gamma$	249	165
$2^3P_0 \longrightarrow$	$\psi(3770)+\gamma$	168	70
$2^1P_1 \longrightarrow$	$\eta_c(1S)+\gamma$	845	86
	$\eta_c'(2S)+\gamma$	293	271
$Z(3930)[2^3P_2] \longrightarrow$	$J/\psi(1S)+\gamma$	744	59
	$\psi(2S)+\gamma$	235	140

Table 6. $E1$ electromagnetic bottomonium widths in keV. The experimental values are from [10].

		Theor.	Expt.
$\Upsilon(2S) \longrightarrow$	$\chi_{b2}(1P)+\gamma$	3.30 ± 0.04	2.21 ± 0.16
	$\chi_{b1}(1P)+\gamma$	3.21 ± 0.04	2.11 ± 0.16
	$\chi_{b0}(1P)+\gamma$	2.10 ± 0.02	1.14 ± 0.16
$\Upsilon(3S) \longrightarrow$	$\chi_{b2}(2P)+\gamma$	3.22	2.95 ± 0.21
	$\chi_{b1}(2P)+\gamma$	2.95	2.71 ± 0.20
	$\chi_{b0}(2P)+\gamma$	1.82	1.26 ± 0.14

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